

Eclipse for Windows

Version 5

Device Stability and The Bilinear Transform

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Device Stability and The Bilinear Transform

The Bilinear Transform

Often it is desirable to represent data in several different graphical forms because of the unique perspective offered by each representation. Typically, there are simple equations which link these graphical forms together. One such equation is known as the bilinear transform (BLT). Its standard form is the ratio of two 1st degree polynomials of the same independent variable:

$$(1) \quad Y = \frac{a_1 X + a_2}{a_3 X + 1}$$

where a_1 , a_2 and a_3 are complex coefficients. In general, BLT's map straight lines and circles in the independent-variable plane onto other circles and/or lines in the dependent-variable plane. It can also be shown that the bilinear transform of a bilinear transform is again bilinear. For instance, given

$$(2) \quad Z = \frac{b_1 Y + b_2}{b_3 Y + 1}$$

and by substituting (1) into (2) we get

$$(3) \quad Z = \frac{c_1 X + c_2}{c_3 X + 1}$$

where

$$c_1 = (a_1 b_1 + b_2 a_3) / (1 + a_2 b_3)$$

$$c_2 = (a_2 b_1 + b_2) / (1 + a_2 b_3)$$

$$c_3 = (a_1 b_3 + a_3) / (1 + a_2 b_3)$$

The most familiar BLT to RF engineers, shown in equation (4), is the one used to map the impedance plane (Z-plane) into the reflection coefficient plane (Γ -plane):

$$(4) \quad \Gamma = \frac{Z - R_0}{Z + R_0}$$

The real constant, R_0 , is known as the normalizing resistance. In this paper, Γ will strictly mean mapping using the form shown in (4). Through comparison with (1), the coefficients for this mapping are seen to be

$$a_1 = \frac{1}{R_0}$$

$$a_2 = -1$$

$$a_3 = \frac{1}{R_0}$$

This BLT maps the entire Right Half Z-plane onto the interior of a unit circle located at the origin as shown in *Figure 1*. The Left Half Z-plane maps onto the exterior region. The resultant mapping is known as the Smith

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Chart. The origin or center of the unit circle represents the normalizing resistance, R_o , and is the intersecting point of the lines $R=R_o$ and $X=0$. The magnitude of the transform defined by (4) can be thought of as the measure or distance of an impedance, Z , from the normalizing resistance. When $Z=R_o$ the magnitude of (4) is zero

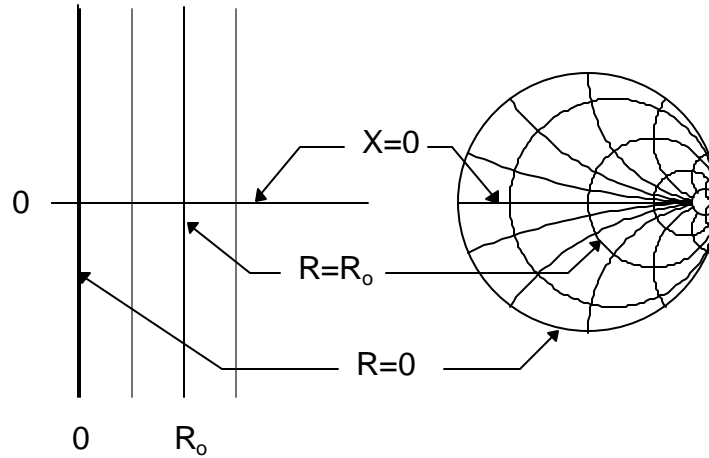


Figure 1 Impedance to Reflection mapping via R_o

Complex Normalizing Impedances

Although we almost always use positive-real values as normalizing impedances, it is just as meaningful to use a complex number as a normalizing impedance as in (5). In doing this, the Right Half Z-plane still maps into the unit circle interior (if $R_o > 0$), but now the circle's center is coincident with the lines $R=R_c$ and $X=X_c$. Note that in *Figure 2* the center of the Smith Chart represents, Z_c , and that the reactance line, $X=0$, has become an arc on the Smith Chart. For the remainder of this paper, Ψ , will be strictly used to indicate mapping using a complex normalizing impedance.

$$(5) \quad \Psi = \frac{Z - Z_c}{Z + Z_c^*}$$

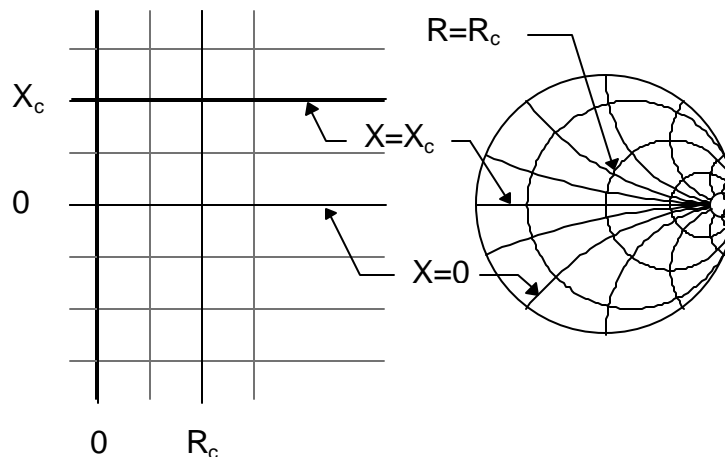


Figure 2 Impedance to Reflection mapping via Z_c

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Often we want to know how well an impedance matches to a given termination or, in other words, how close the impedance is to the termination's complex conjugate; thus, if the terminating impedance is Z_T , then the normalizing impedance is $\text{conj}(Z_T)$ and the correct BLT, after substitution into (5), is given by (6). To compare the input impedance of some device to a complex reference impedance (perhaps the output impedance of a prior stage), the center of the Smith Chart should be the conjugate of the driving impedance.

$$(6) \quad \Psi = \frac{Z - Z_T^*}{Z + Z_T}$$

Stability Analysis

For unconditional stability, a 2-port network must exhibit positive resistance at each port for all possible passive load values at the opposite port; otherwise, the network may oscillate. Even though a network may exhibit negative input resistance for some specific load does not mean that the circuit will necessarily oscillate. For example, if the net series resistance at the input port ($R_{\text{source}} + R_{\text{in}}$) is positive, then the circuit can not oscillate due to a negative R_{in} .

One method of stability analysis begins with an equation that relates the input reflection coefficient to the load reflection coefficient using the 2-port S-parameters of the network.

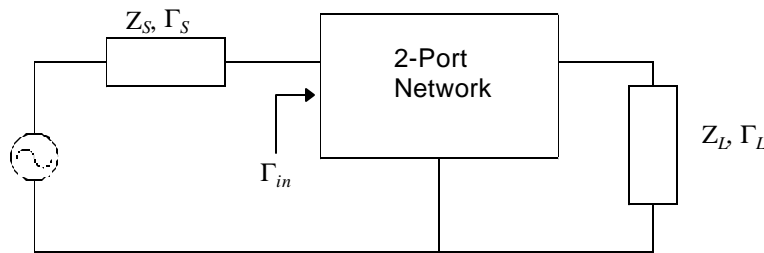


Figure 3 2-Port Network

$$(7) \quad \Gamma_{in} = \frac{S_{11} - \Delta \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\text{where } \Delta = S_{11} S_{22} - S_{21} S_{12}$$

Note that equation (7) is a BLT of the independent variable, Γ_L , where

$$\begin{aligned} a_1 &= -\Delta \\ a_2 &= S_{11} \\ a_3 &= -S_{22} \end{aligned}$$

If Γ_L is replaced with the normalized impedance transform

$$\Gamma_L = \frac{Z_L - 1}{Z_L + 1}$$

then (7) can be written as

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$$(8) \quad \Gamma_{in} = \frac{b_1 Z_L + b_2}{b_3 Z_L + 1}$$

which is now a BLT of the variable, Z_L , with the coefficients in terms of the original S-parameters.

$$b_1 = \frac{S_{11} - \Delta}{1 + S_{22}}$$

$$b_2 = \frac{S_{11} + \Delta}{1 + S_{22}}$$

$$b_3 = \frac{1 - S_{22}}{1 + S_{22}}$$

Equation (6) can be written in the generalized form

$$(9) \quad \Gamma_{in} = T + R \frac{Z_L - Z_c}{Z_L + Z_c^*}$$

where T, R and Z_c are complex constants expressed in terms of the coefficients, b_1 , b_2 , and b_3 .

$$T = \frac{b_2 b_3^* + b_1}{2 \operatorname{Re}(b_3)}$$

$$R = \frac{b_1}{b_3} - T$$

$$Z_c = \frac{1}{b_3^*}$$

The form of equation (9) is extremely useful and can be intuitively described as follows:

- the equation contains a BLT in the independent variable Z_L ; thus the Right Half Z-plane is mapped to a unit circle at the origin with the center corresponding to the complex normalizing impedance, Z_c .
- the unit circle is then multiplied by the complex constant, R, which scales and rotates the circle by m and q respectively.

$$R = m \cdot e^{jq}$$

- lastly, a constant, T, is added which translates the scaled circle's origin to a new location.

Visually, a particular BLT might transform the Right-Half Z_L -plane as shown in *Figure 4* where the load plane is represented as a Smith Chart having been scaled, rotated and translated. Circles C_0 and C_1 represent the Γ_{in} -plane and mapped Z_L -plane respectively. If the real part of the complex normalizing impedance is positive, then the interior of C_1 represents all possible passive load values. There are no passive load values

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in this example (assuming $\text{Re}(Z_C) > 0$) that will result in $\Gamma_{in} = 0$ because C_1 does not encompass C_0 's origin. In this article the unit circle will be referred to as the Reference Plane.

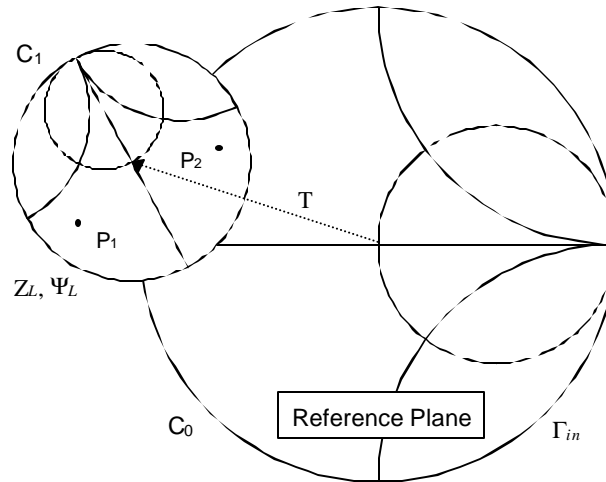


Figure 4 Overlay of Input and Load Planes

Perhaps the most interesting property of this mapping is the one-to-one correspondence between the two planes established by C_0 and C_1 ; specifically, it is an overlay of the load plane and the load plane as seen through the device (i.e., the input impedance). For instance, P_1 in *Figure 4* represents an impedance in the Z_L -plane being appropriately mapped into the circle C_1 by the complex normalizing impedance, Z_C ; however, P_1 also represents the corresponding value of Γ_{in} relative to the circle, C_0 . In this particular example, P_1 is outside the Reference Plane, C_0 , and therefore has a negative resistance. P_2 is in the interior of both planes. The real parts of the impedances represented by P_1 and P_2 (relative to C_1) will have the same sign as the real part of the normalizing impedance, Z_C .

BLT Examples

Using S-parameter data for a 2N6604 transistor at 200 MHz several useful transformations will be demonstrated.

$$\begin{aligned} S_{11} &= .53 @ -145 \text{ (Mag @ angle)} \\ S_{21} &= 19.35 @ 114 \\ S_{12} &= .022 @ 49 \\ S_{22} &= .46 @ -57 \end{aligned}$$

Case 1 - Mapping the Ψ_L Plane into the Γ_{in} Reference Plane

$$\Gamma_{in} = \frac{b_1 Z_L + b_2}{b_3 Z_L + 1}$$

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This mapping follows the discussion relative to *Figure 4* and therefore allows a termination plane, Ψ_L , to be viewed simultaneously with the corresponding Γ_{in} reference plane. Equation (8) is the proper transform and is repeated here for convenience. Given the above S-parameters, equation (9) yields the following with the results graphically shown in *Figure 5*.

$$\begin{aligned} T &= 0.7777 @ -143.4 \\ R &= 0.54 @ 108.5 \\ Z_C &= 55.48 + j54.3 \end{aligned}$$

Since the real part of Z_C is positive, the Right-Half Z_L -plane maps into C_0 's interior. As can be seen, there are impedance values in the right and left Z_L -planes that lie in the exterior of C_0 . P_1 and P_2 are such values. Z_{in} will have a negative real part for both of these values. Similarly, there are values in the right and left Z_L -plane (P_3 and P_4) that lie in the interior of C_0 which have positive real parts for Z_{in} . As there are passive load values within C_1 that correspond to input negative resistance relative to C_0 (e.g., P_2), the device is potentially unstable at this frequency. The complex normalizing impedance for the Load plane is $55.48 + j54.3$ which means that the $X=0$ reactance line is an arc as shown in *Figure 5*. The capacitive region of the load plane is the shaded area.

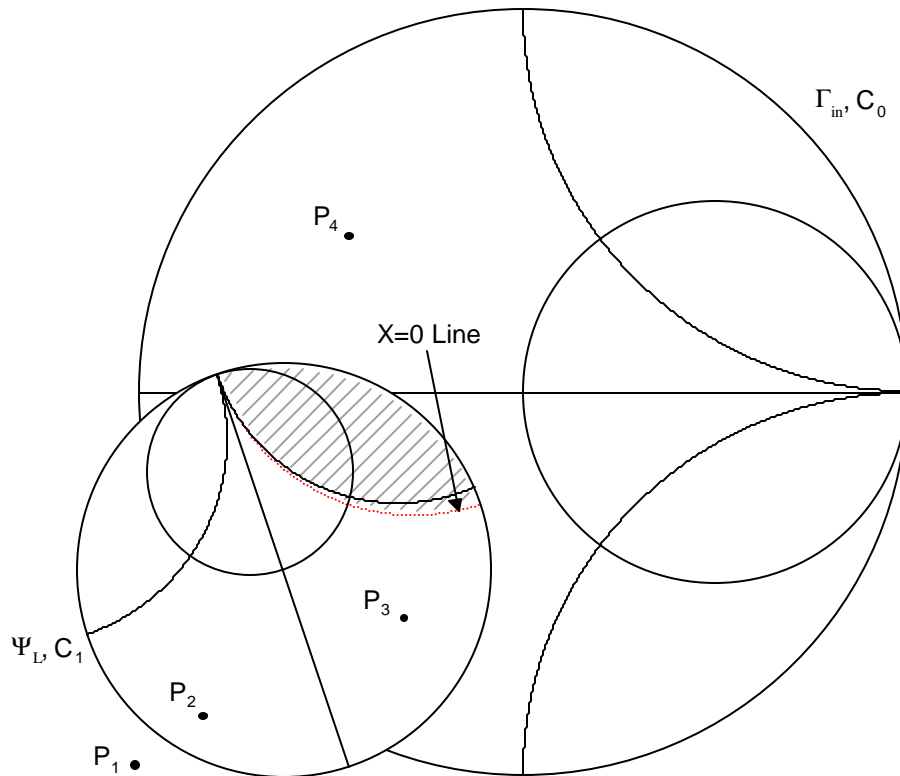


Figure 5 Mapping, Case 1

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Case 2 - Mapping the Ψ_L -plane into the Z_{in} -plane

This mapping allows the load plane, Ψ_L , to be viewed simultaneously with the corresponding Z_{in} reference plane. The uniqueness of this mapping is that one entire half of the load plane (the positive half in this example) is confined to the interior of the circle while displayed in the usual Cartesian impedance plane. This transform is derived by substituting

$$\Gamma_{in} = \frac{Z_{in} - 1}{Z_{in} + 1}$$

into equation (8) and rearranging to obtain

$$Z_{in} = \frac{(b_3 + b_1)Z_L + 1 + b_2}{(b_3 - b_1)Z_L + 1 - b_2} = \frac{c_1 Z_L + c_2}{c_3 Z_L + 1}$$

where

$$c_1 = \frac{1 + S_{11} - S_{22} - \Delta}{1 - S_{11} + S_{22} - \Delta}$$

$$c_2 = \frac{1 + S_{11} + S_{22} + \Delta}{1 - S_{11} + S_{22} - \Delta}$$

$$c_3 = \frac{1 - S_{11} - S_{22} + \Delta}{1 - S_{11} + S_{22} - \Delta}$$

For this case

$$\begin{aligned} T &= 0.4504 @ -53.48 \\ R &= 0.4215 @ 111.6 \\ Z_C &= 44.04 + j22.97 \end{aligned}$$

As might be expected, the circle crosses into the Left-Half Z_{in} -plane indicating that a region of negative resistance is possible for passive loads. Since Z_C has a positive real part, the Right-Half Z_L -plane maps into C_1 's interior. Note that even though Z_C is different for case 1 and case 2, the values of Z_L that lie in the unstable region must necessarily be the same in both cases.

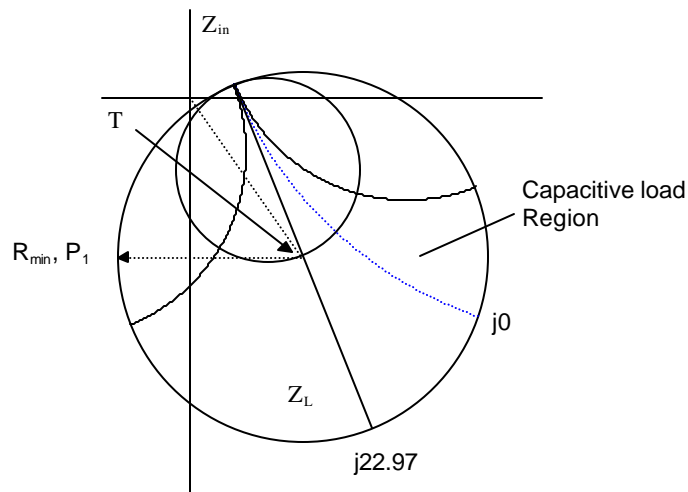


Figure 6 Mapping, Case 2

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From this particular mapping it is easy to determine impedance extremes and their corresponding load values. For instance, P_1 represents the minimum possible real input resistance that any passive load will produce. In the Z_{in} -plane this value by inspection is

$$R_{\min} = 50(\operatorname{Re}(T) - |R|) = -7.67 \text{ Ohms}^1$$

Determining the corresponding load value is simple since it resides on the circumference of the circle. Relative to the transformed load plane, the load reflection coefficient is

$$\Gamma_L = 1 @ (180 - \operatorname{ang}(R)) = 1 @ 68.4$$

Solving for Z_L in equation (5) provides the corresponding reactance value by denormalizing the reflection coefficient and is given by

$$Z_L = \frac{Z_C + Z_C^* \Gamma_L}{1 - \Gamma_L} = j87.8 \text{ Ohms}$$

In *Figure 6*, it can be seen that the complex normalizing impedance, Z_C , has caused the capacitive portion of the Z_L -plane to be compressed into a small region of C_1 . This mapping can be useful in oscillator design where it is important to control the magnitude of the input negative resistance.

Case 3 - Mapping the Z_{in} -plane into the Γ_L Reference plane

This mapping basically reverses the case 1 mapping by having the termination reflection coefficient or load as the reference plane. The results of this mapping yield the “traditional” stability circles defined in most text books. This transform is derived by substituting

$$\Gamma_{in} = \frac{Z_{in} - 1}{Z_{in} + 1} \quad \text{and} \quad Z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

into equation (8) and rearranging to obtain

$$\Gamma_L = \frac{d_1 Z_{in} + d_2}{d_3 Z_{in} + 1}$$

where

$$d_1 = \frac{S_{11} - 1}{1 + S_{22}}$$

$$d_2 = \frac{S_{11} + 1}{1 + S_{22}}$$

$$d_3 = \frac{1 - S_{22}}{1 + S_{22}}$$

¹ Multiplication by 50 occurs because T and R were derived from 50 ohm S-parameters.

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and

$$\begin{aligned}T &= 3.128 @ 54.97 \\R &= 2.395 @ 140.7 \\Z_C &= -68.78 + j47.64\end{aligned}$$

The results are shown in *Figure 7*.

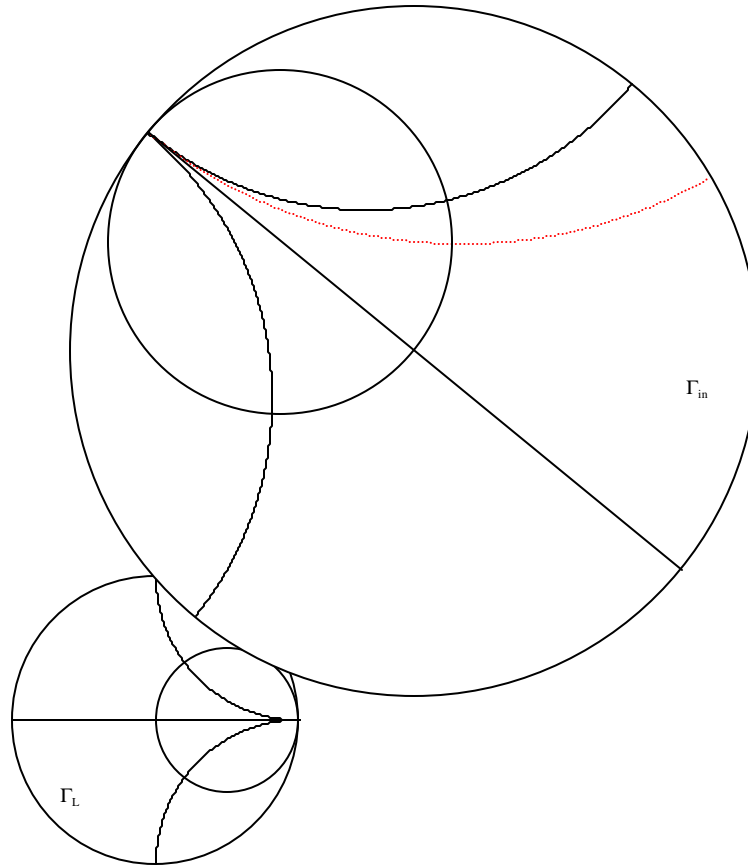


Figure 7 Mapping, Case 3

It should be noted that the three cases show different views of the same information, each view having its own advantages.

Stabilizing a Potentially Unstable Device

The thorough design of any amplifier requires that attention be given to device stability for a broad range of conditions which should include loading, aging and temperature effects. A given design may work fine

- at room temperature but oscillate at cold temperatures where device gain may have risen
- into a 50 ohm system (such as a network analyzer) but fails when loaded with a real antenna

Any analysis that does not include the effects of all potential operating conditions proceeds to production with dormant consequences begging to inflict personal embarrassment.

Device Stability and The Bilinear Transform

The first step in the design process is to determine if there is any potential instability in the chosen active device (including biasing). Since a given device can be potentially unstable at some frequencies yet stable at others, it is necessary to perform a broad band analysis. An RF device can look great at 1 GHz in the passband region but exhibit a 200 Mhz oscillation because of inherent instability at lower frequencies. Further, a device's parameters may predict unconditional stability over all frequencies yet become unstable due to poor grounding techniques when actually implemented. Of course, this would indicate the model was not correct in the first place since actual parasitics were not adequately accounted for. A broad band analysis using the BLT techniques previously discussed is certainly possible but can become visually cluttered if more than a few frequency points (stability circles) are graphed. For a quick look at stability as a "go/no go" situation a numerical stability factor can be more useful. One such factor has been defined² as

$$(10) \quad U = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta S_{22}^*| + |S_{21}S_{12}|} > 1$$

The utility of this equation is that it completely defines the stability conditions of a 2-port. If the value is greater than or equal to one then the 2port is unconditionally stable (i.e., no passive load can cause oscillations). Most other published criteria requires two or more equations to predict stability.

Let's apply the techniques discussed so far to an interesting MMIC from Hewlett-Packard, the INA10386. The published s-parameters for the INA10386 predict unconditional stability ($U > 1$) from 50 to 4000 Mhz; however, HP's own appnote says, "excellent grounding is critical when using INA MMICs"³. How critical? If as little as 0.25 nH is added in series with the emitter leads to ground, the factor U drops below 1 for a significant portion of the high-frequency range. This means there is a range of passive terminations that will create negative resistance at the opposite port. The size and number of grounding vias for this device is obviously very important. In concert with this, HP states that INA "designs should be done on the thinnest practical substrate. The parasitic inductance of a pair of via holes passing through .032 inch pc board is approximately 0.1 nH, while that of a pair of via holes passing through .062 thick board is close to .5 nH. HP does not recommend using INA family MMICs on boards thicker than 32 mils". Their suggested layout on .032 inch substrate includes six grounding vias which would be approximately 0.13 nH on 0.062 substrate — the middle trace in *Figure 8* and *Figure 9*! Note the gain peaking in *Figure 9*. The unstable region in this scenario covers the entire cellular band.

Stability circles are shown in *Figure 10* at 0.7, 1.0 and 2.0 Ghz. A small dot adjacent to each circle indicates the region of instability (exterior in each case). To guarantee unconditional stability, the load impedance at each frequency must lie in the interior of the corresponding circle.

² M.L. Edwards, "A New Criterion for Linear 2-port Stability Using a Single Geometrically Derived Parameter", IEEE Trans. Vol. 40. No. 12., December 1992.

³ HP Communications Components, Designer's Catalog 1993, p 8-61

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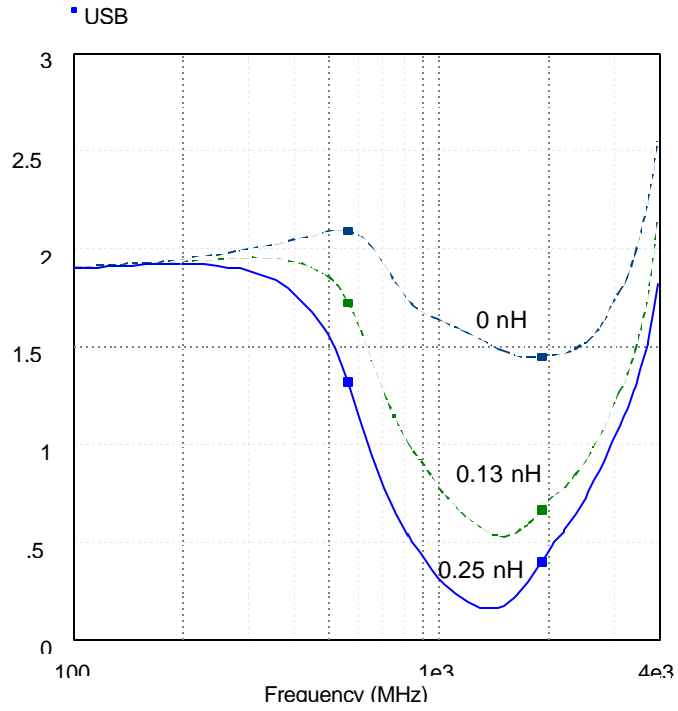


Figure 8 INA10386 Device Stability

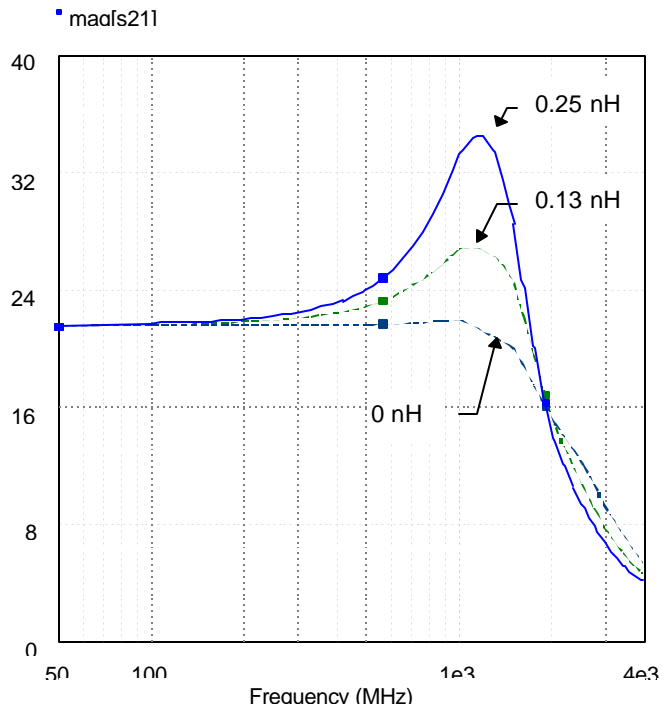


Figure 9 INA10386 Gain Response

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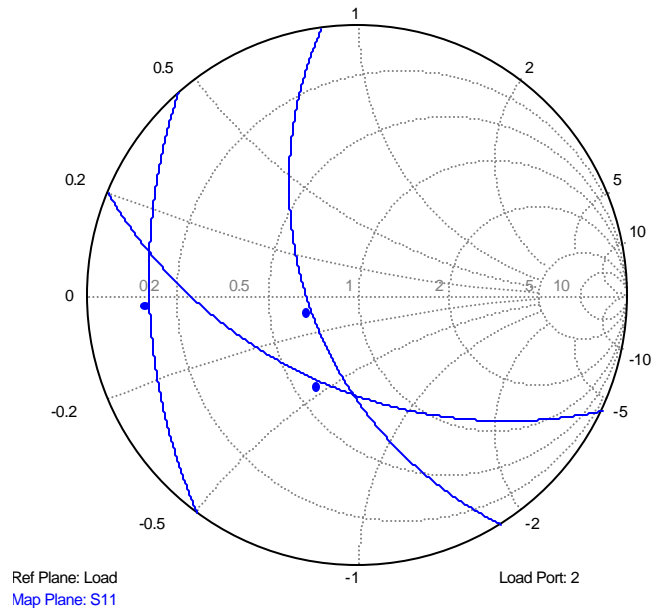


Figure 10 INA10386 Stability Circles, .25 nH lead Inductance

Stability Using Feedback

Stabilizing a device through feedback (series, shunt or both) in effect changes the S-parameters of the composite circuit such that there are no regions of instability — maybe. Feedback can improve stability in one frequency range while creating instability in another range. The INA10386 example given above actually creates instability through unintentional feedback, i.e., poor grounding. Broad-band analysis is necessary to avoid such cases. Purely reactive feedback eliminates losses in the feedback elements themselves. This is especially important in applications where output intercept point and/or low-noise are critical specifications.

Resistive loading

A simple approach to device stabilization is to resistively load one or both of its ports. Besides being simple, resistive loading is inherently broad-band. The disadvantages of this approach is reduced available power, lower output intercept-point and possibly higher noise figure. In addition, resistive loading can not always provide stability as will be explained later.

Basically, all forms of resistive loading constrain the external load, as seen by the device, to a smaller region of the reflection plane. Given this fact, it is possible that a series or parallel resistor can be chosen to keep the effective load out of the unstable region. For instance, in *Figure 11* a series 22.3 ohm resistor or shunt 142 ohm resistor at port 2 is sufficient to keep effective load values (due to any passive termination) from extending into the unstable region. A re-evaluation of the circuit with the loading resistance included would indicate no unstable region, i.e., the load plane and the mapped plane boundaries no longer intersect. *Figure 12* shows the load plane (including loading resistance) mapped onto the S_{11} plane. Note that all passive load values now fall in the positive real region of the input plane due to the resistive loading.

There are cases, however, where a specific form of resistive loading can not be used to achieve unconditional stability as shown in *Figure 13*. Although a 75.3 ohm series resistor at port 2 can provide stability, there is no similar parallel value at that port. In fact, a small enough parallel resistance (38.1 ohms in this case) would actually insure that the effective load is *always* in the unstable region (also indicated in the illustration). The mapping in *Figure 13* shows that a 50 ohm load would create an almost purely reactive input impedance. The suggested bias current and voltage for the INA10386 is 45 mA and 6 V respectively.

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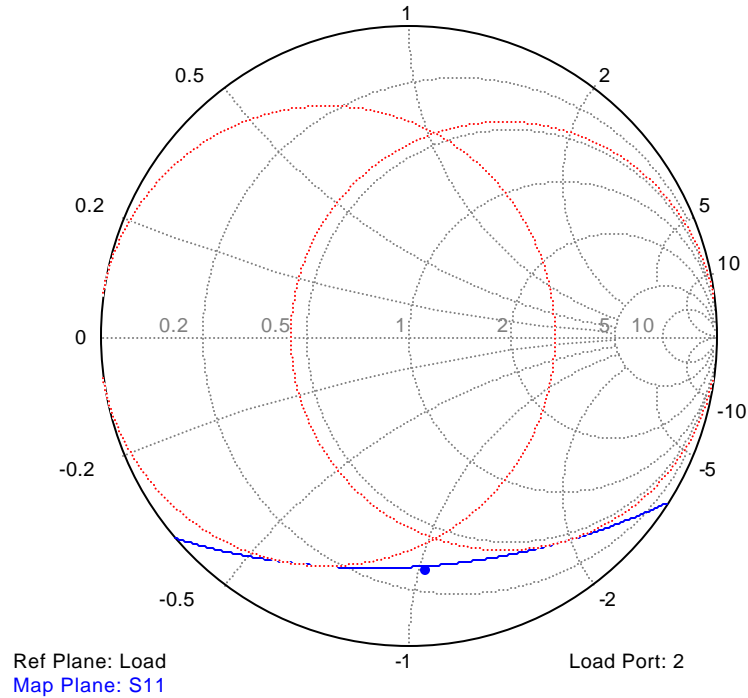


Figure 11 Resistive loading necessary at 2.6 GHz (.25 nH emitter inductance)

If a 9 V supply is used, a biasing resistance of approximately 68 ohms would be required. For this loading an RFC choke in series with the bias resistor would probably be used as well. But if the choke is past self-resonance at 1.5 GHz, the combination of the bias resistor and 50 ohm termination would likely fall into the unstable region resulting in a corresponding negative input resistance (about -12 ohms).

Summary

Bilinear transforms permit simultaneous viewing of load and input impedance (or reflection) planes. Regions of instability, impedance extremes and general trends can easily be determined through these mappings. *Table 1* gives several bilinear forms and their necessary coefficients.

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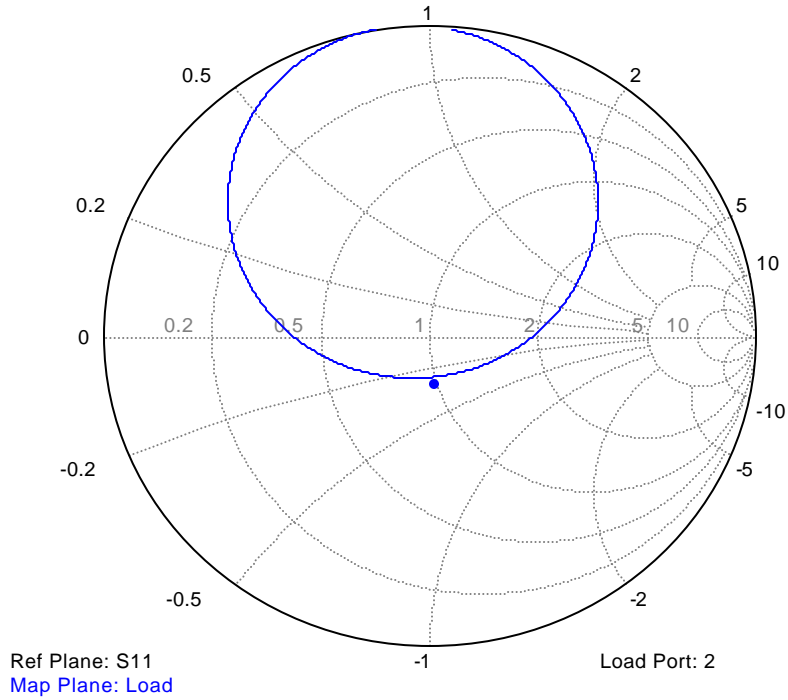


Figure 12 Resistive loading included (2.6 GHz, .25 nH emitter inductance)

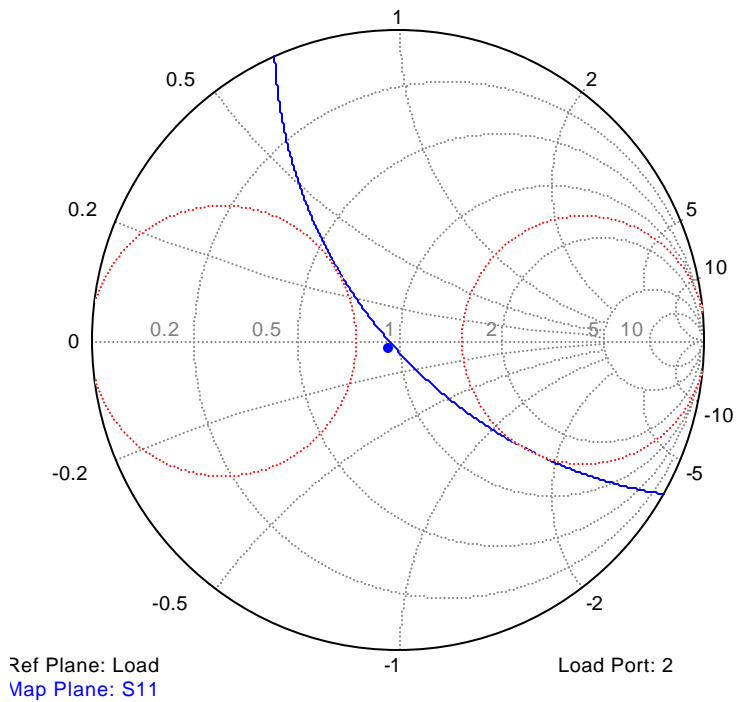


Figure 13 Resistive loading necessary at 1.5 GHz (.25 nH emitter inductance)

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Given:

$$\Delta = S_{11}S_{22} - S_{21}S_{12}$$

$$a_1 = -\Delta,$$

$$a_2 = S_{11},$$

$$a_3 = -S_{22}$$

$$b_1 = \frac{S_{11} - \Delta}{1 + S_{22}},$$

$$b_2 = \frac{S_{11} + \Delta}{1 + S_{22}},$$

$$b_3 = \frac{1 - S_{11}}{1 + S_{22}}$$

$$c_1 = \frac{S_{11} - 1}{S_{22} + \Delta},$$

$$c_2 = \frac{S_{11} + 1}{S_{22} + \Delta},$$

$$c_3 = \frac{\Delta - S_{22}}{S_{22} + \Delta}$$

and $Y = \frac{k_1 X + k_2}{k_3 X + 1}$, where Y is the reference plane variable and X is the mapped variable.

Equation	Y (reference)	X (map)	k_1	k_2	k_3
(1)	Γ_{in}	Γ_L	a_1	a_2	a_3
(2)	Γ_L	Γ_{in}	$1/a_1$	$-a_2/a_1$	$-a_3/a_2$
(3)	Γ_{in}	Z_L	b_1	b_2	b_3
(4)	Z_{in}	Z_L	$(b_3 + b_1)/(1 - b_2)$	$(1 + b_2)/(1 - b_2)$	$(b_3 - b_1)/(1 - b_2)$
(5)	Γ_L	Z_{in}	c_1	c_2	c_3

then,

$$Z_C = \frac{1}{\bar{k}_3} \quad \text{Complex Normalizing Impedance}$$

$$T = \frac{k_2 \bar{k}_3 + k_1}{2 \cdot \text{Re}(k_3)} \quad \text{Translation factor}$$

$$R = \frac{k_1}{k_3} - T \quad \text{Scaling \& Rotation factor}$$

$$\Psi = \frac{Z - Z_C}{Z + \bar{Z}_C}, \quad Z = \frac{Z_C + \bar{Z}_C \Psi}{1 - \Psi}$$

Table 1 Coefficients for Various BLTs